Section – A

Question number 1 to 10 carry 1 mark each. For each of the questions 1 to 10. Four alternative choice have been provided, of which only one is correct. Select the correct choice.

1. The roots of the equation $x^2 + x - p(p+1) = 0$, where p is a constant, are

(A) p, p+1
(B) – p, p+1
(C) p, – p (p+1)
(D) – p. – (p+1)

Ans. (C) p, – p (p+1)

2. In an AP, if d = -2, n = 5 and a_n = 0, the value of a is

(A) 10
(B) 5
(C) – 8
(D) 8

Ans. (D) 8

3. In fig. 1, O is the centre of a circle. AB is chord and AT is the tangent at A. If ∠AOB = 100°, then ∠ BAT is equal to

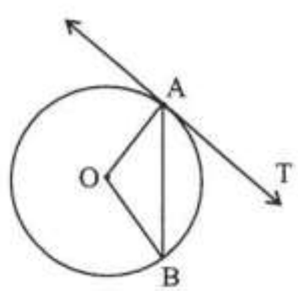


Fig.1

(A) 100°
(B) 40°
(C) 50°
(D) 90°

Ans. (C) 50°

4. In fig 2, PA and PB are tangents to the circle with centre O. If ∠APB = 60°, then ∠OAB is

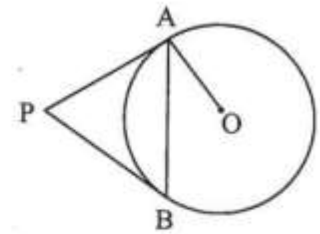


Fig. 2

(A) 30°
(B) 60°
(C) 90°
(D) 15°

Ans. (A) 30°

5. The radii of two circles 4 cm and 3 cm respectively. The diameter of the circle having are equal to the areas of the to circle (in cm) is

(A) 5
(B) 7
(C) 10
(D) 10

Ans. (C) 10

6. A sphere of diameter 18 cm id dropped into a cylindrical vessel of diameters 36 cm, partly filled with water. If the sphere is completely submerged, then the water level rises (in cm) by

(A) 3
(B) 4
(C) 5
(D) 6

Ans. (A) 3

7. The angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of the tower is 45°. The height of the tower (in metres) is

(A) 15
(B) 30
(C) $30\sqrt{3}$
(D) $10\sqrt{3}$

Ans. (B) 30

8. The point P which divides the line segment joining the points A (2, -5) and B (5, 2) in the ratio 2:3 lies in the quadrant.

(A) I
(B) II
(C) III
(D) IV

Ans. (D) IV

9. The mid-point of segment AB is the point P (0, 4). If the coordinates of B are (-2, 3) then the coordinates of A are

(A) (2, 5)
(B) (-2, -5)
(C) (2, 9)
(D) (-2, 11)

Ans. (A) (2, 5)

10. Which of the following cannot to the probability of an event?

(A) 1.5
(B) $\frac{3}{5}$
(C) 25%
(D) 0.3

Ans. (A) 1.5

Section B

Question Number 11 to 18 carry 2 marks each.

11. Find the value of q so that the quadratic equation px(x – 3) + 9 = 0 has two equal roots.

Ans. For roots to be equal $b^2 - 4ac = 0$

$px^2 - px + 9 = 0$

$\Rightarrow 9p^2 - 36p = 0 \Rightarrow 4(p = 0 \text{ rejected})$

12. Find weather – 150 is a term of the AP 17, 12, 7, 2, ….?

Ans. Here a = 17, d = - 5, t_n = - 150

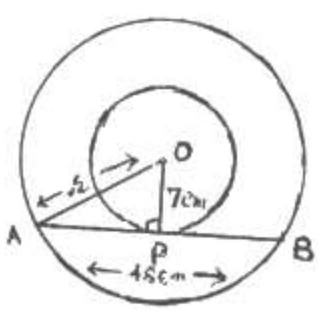
$\Rightarrow -150 = 17 + (n-1)(-5)$

$\Rightarrow \frac{-167}{-5} + 1 = n$, which is not a natural number

∴ - 150 is not a term of the A.P.

13. Two concentric circles are of radii 7 cm anc r respectively, where r>7. A chord of the charger circle, of length 48 cm, touches the smaller circle. Find the value of r.

Ans.



Here OP = 7 cm, OA = r cm

AP = $\frac{48}{2}cm$ or 24 cm

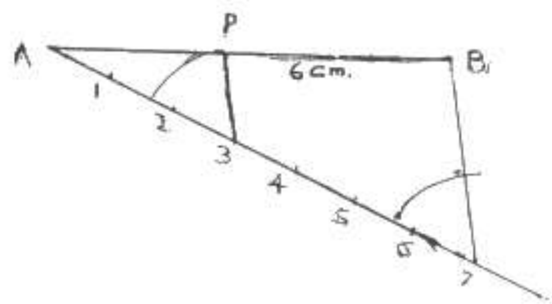
$\therefore r^2 = OP^2 + AP^2 = 24^2 + 7^2 = 625$

$\Rightarrow r = 25$ cm.

14. Draw a line segment of length 6cm. Using compassed and ruler, find a point P on it which divided it in the ratio 3:4.

Ans.

Correct construction



15. In Fig. 3, APB and CQD are semi circles of diameter 7 cm each, while ARC and BSD are Semi-circle of diameter 14 cm each. Find the perimeter of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$

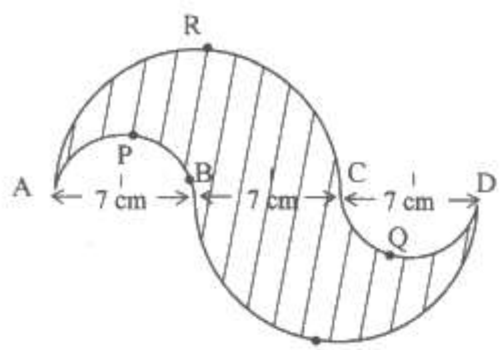


Fig. 3

OR

Find the area of a quadrant of a circle, where the circumference of circle is 144 cm. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Ans. Perimeter of two bigger semicircles

$$= 2\left(\frac{22}{7} \times 7\right) cm = 44 cm$$

Perimeter of two smaller semicircles = $2 \times \frac{22}{7} \times \frac{7}{2} = 22 cm$

∴ Total perimeter = (44 + 22) cm or 66 cm

OR

$$2\pi r = 44 \Rightarrow 2 \times \frac{22}{7} \times r = 44 \Rightarrow r = 7 cm$$

∴Area of quadrant of radius r = $\left(\frac{1}{4} \times \frac{22}{7} \times 7 \times 7\right) cm^2$

= 38.5 cm^2

16. Two cubes, each of side 4 cm are joined end to end. Find the surface area of the resulting cuboid.

Ans. Dimension of resulting cuboid

Length = 8cm, Breath =-Height = 4cm

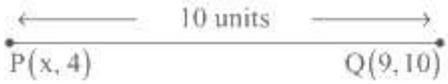
∴Surface area of cuboid

$$= 2[8 \times 4 + 4 \times 4 + 8 \times 4] cm^2$$

= 160 cm^2

17. Find the value(s) of x for which the distant between the points P (x, 4) and Q (9, 10) is 10 units.

Ans.



PQ2 = $(x-9)^2 + (4-10)^2 = 100$

$\Rightarrow x^2 - 18x + 17 = 0$

$(x-1)(x-17) = 0$

X = 1, 17

18. A coin is tossed two times. Find the probability of getting at least one head.

Ans. Total outcomes are HH, HT, TH, TT

P (at least one Head) 1 – P (no Head)

$= 1 - \frac{1}{4} = \frac{3}{4}$

Section C

Question numbers 19 to 28 carry 3 marks each.

19. Find the roots of the following quadratic equation:

$2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$

Ans.

$2\sqrt{3}\ x^2 - 5x + \sqrt{3} = 0$

$2\sqrt{3}\ x^2 - 2x - 3x + \sqrt{3} = 0$

$\Rightarrow\ 2x(\sqrt{3}\ x - 1) - \sqrt{3}(\sqrt{3}x - 1) = 0$

$(2x - \sqrt{3})(\sqrt{3}x - 1) = 0$

$\Rightarrow\ x = \frac{1}{\sqrt{3}}, \frac{\sqrt{3}}{2}$

$\therefore$ The roots are $\frac{1}{\sqrt{3}}, \frac{\sqrt{3}}{2}$

20. Find the value of the middle term of the following AP:
- 6, - 2, 2, ………, 58.

OR

Determine the AP whose fourth term is 18 and the difference of the ninth term from the fifteenth term is 30.

Ans. Here a = - 6, d = 4, a_n = 58
$\therefore$ 58 = - 6 +(n -1) 4
n= 17
$\therefore$ Middle term is 9^{th}
$a_9 = a + 8d = -6 + 32 = 26$

OR

$a_4 = 18$ and $a_{15} - a_9 = 30$
$\Rightarrow$ a + 3d = 18 and (a+14d) – (a+8d) = 30
$\Rightarrow$ d = 5
$\Rightarrow$ a + 15 = 18 $\Rightarrow$ a = 3
$\therefore$ AP is 3, 8, 13, 18, ………

21. In Fig. 4, a triangle ABC is drawn to circumscribe a circle of radius 2 cm such that the segments BD and BC is divided by the point of contact D are lengths 4 cm and 3 cm respectively. If area of $\Delta ABC = 21 cm^2$, then find the length of sides of AB and AC.

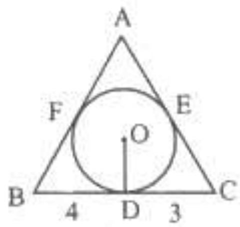


Fig. 4

Ans. Let AF be equal to x = AE = x

Also, BF = BD = 4 cm, CD = CE = 3cm

$\therefore$ Semi-perimeter of ΔABC is (7+x)

$\Rightarrow$ (7+x)2 = 21 $\Rightarrow$ x = 3.5

22. Draw a triangle ABC in which AB = 5 cm, BC = 6 cm and $\angle$ABC = 60°.

The construct a triangle whose sides are $\frac{5}{7}$ times the corresponding sides of ΔABC.

Ans.

23. Find the area of the major segment APB, in Fig. 5 of a circle of radius 35 cm and $\angle ABC = 90°$. $\left[\text{Use } \pi = \frac{22}{7}\right]$

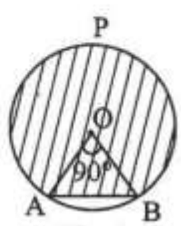


Fig. 5

Ans. Area of minor segment = Area of (sector OAB – ΔOAB)

$$= \left[\frac{90}{360} \times \frac{22}{7} \times (35)^2 - \frac{1}{2} \times 35 \times 35\right] cm^2$$

$= \frac{1925}{2} - \frac{1225}{2}$ Or 350 cm^2.

$\therefore$ Area of major segment $= \left(\frac{22}{7} \times 35 \times 35 - 350\right) cm^2$

= 3500 cm^2.

24. The radii of the circular ends of a bucket of height 15 cm are 14 cm and r cm (r<14cm). If the value of bucket is 5390 cm^3, then find the value of r. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Ans. Volume of bucket $= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

$$5390 = \frac{1}{3} \times \frac{22}{7} \times 15[14^2 + r^2 + 14r]$$

$$49 \times 7 = (196 + (r^2 + 14r)$$

$$\Rightarrow r^2 + 14r - 147 = 0$$

$$\Rightarrow (r + 21)(r - 7) = 0 \Rightarrow r = 7cm$$

25. Two dice are rolled once. Find the probability of getting such number on two dice, whose product is a perfect square.

OR

A game consists of tossing a coin 3 times and noting its outcome each time. Hanif wins if he gets three heads or three tails, and loses heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

Ans. Total number of possible outcomes = 36

The number of cases favorable to the event

"Product is a perfect square" are 8

[(1,1), (2,2), (3, 3), (4, 4), (5, 5), (6,6), (1,4), (4,1)]

$\therefore$ Required probability $= \frac{8}{36} = \frac{2}{9}$

OR

Total number of outcomes are {HHH, HHT, HTH, THH, THT, TTH, TTT} 8

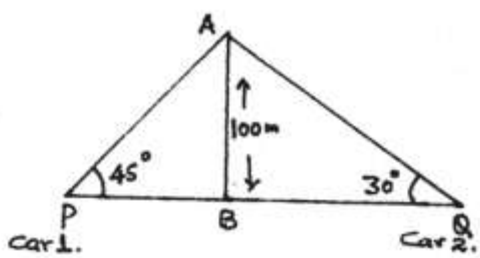
∴No. of cases favorable to winning = 2

No. of cases when game is lost = 6

∴Required probability $= \frac{8}{36} = \frac{2}{9}$

26. From the top of a tower 100 m high, a man observes two cars on the opposite sides of the tower with angles of depression 30° and 45° respectively. Find the distance between the cars. [Use $\sqrt{3} = 1.73$]

Ans.



(i) $\frac{AB}{PB} = \tan 45^\circ = 1$

$\Rightarrow PB = 100m$

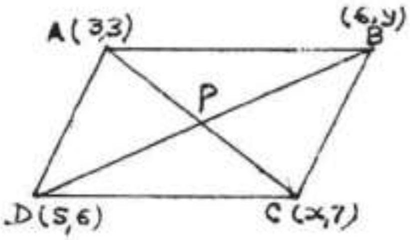
(ii) Again $\frac{AB}{BQ} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

$\Rightarrow BQ = 100\sqrt{3}$ Or 173 cm

∴ PQ = (100 + 173) m or 273 m

27. If (3,3), (6, y), and (5, 6) are the vertices of a parallelogram taken in order, find the value of x and y.

Ans.



In the figure, ABCD is a ||^{gm}

Diagonals of a ||^{gm} bisect each other

Let p be the point of intersection

Mid – point of AC = Mid-point of BD

$$\left(\frac{x+3}{2}, 5\right) = \left(\frac{11}{2}, \frac{y+6}{2}\right)$$

$$\Rightarrow x = 8, y = 4$$

28. If two vertices of an equilateral triangle are (3, 0) and (6, 0), find the third vertex.

OR

Find the value of k, if the points P(5, 4), Q(7, k) and R(9, -2) are collinear.

Ans. ABC is an equilateral triangle of side 3 units

$$\therefore \ AB^2 = AC^2 = 9$$

$$\Rightarrow (x-3)^2 + y^2 = (x-6)^2 + y^2$$

$$\Rightarrow 6x = 27, \Rightarrow x = \frac{9}{2}$$

$$\therefore \left(\frac{9}{2} - 3\right)^2 + y^2 = 9 \Rightarrow y^2 = \frac{27}{4} \Rightarrow y = \pm\frac{3\sqrt{3}}{2}$$

$$\therefore \text{ Coordinates of A are } \left(\frac{9}{2}, \pm\frac{3\sqrt{3}}{2}\right)$$

OR

For P, Q and R to be collinear, area (ΔPQR) = 0

$\Rightarrow$ 5(k + 2) + 7(-2-4) + 9(4-k) = 0

$\Rightarrow$ 5k + 10 – 42 + 36 – 9k = 0

$\Rightarrow$ 4k = 4 $\Rightarrow$ k = 1

Section D

Question numbers 29 to 34 carry 4 marks each.

29. A motor boat whose speed is 20Km/h in still water, takes 1 hour more to go 48 km upstream than to return downstream to the same spot. Find the speed of the stream.

OR

Find the roots of equation $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}, x \neq -4, 7$

Ans. Let the speed of stream be x km/hour

Speed downstream = (20+x) km/hr.

$$\therefore \frac{48}{20-x}-\frac{48}{20+x}=1$$

$$\text{or } 48[20+x-20+x]=400-x^2$$

$$\Rightarrow x^2+96x-400=0$$

$$(x+100)(x-4)=0$$

$$\Rightarrow x=4$$

i.e. speed of stream is 4km/hour

OR

$$\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}$$

Or $30(x-7-x-4)=11(x^2-3x-28)$

$X^2-3x+2=0$

X = 1, x = 2

30. If the sum of first 4 terms of an AP is 40 and that of first 14 terms is 280, find the sum of its first n terms.

OR

Find the sum of the first positive integers divisible of by 6.

Ans.

$$40 = S_4 = \frac{4}{2}[2a + 3d] \Rightarrow 2a + 3d = 20 \text{ ....(i)}$$

$$280 = S_{14} = 7[2a + 13d] \Rightarrow 2a + 13d = 40 \text{ ....(ii)}$$

From (i) and (ii), a = 7, d = 2

$$S_n = \frac{n}{2}[14 + (n-1)2] = n[7 + (n-1)]$$

$= n^2 + 6n$

OR

6, 12, 18, ….., 30 terms

$$S_{30} = \frac{30}{2}[12 + 29 \times 6]$$

$$S_{30} = 15[186] = 2790$$

31. Prove that length of tangents drawn from a external point to a circle are equal.

32. In Fig. 6, arcs are drawn by taking vertices A, B and C of an equilateral ABC of side 14 cm as centre to intersect the sides BC, CA and AB at this respective mid – point D, E and F. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7} \text{ and } \sqrt{3} = 1.73\right]$

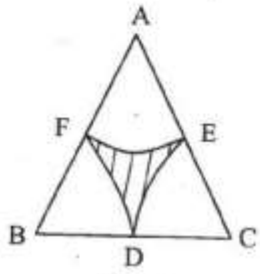


Fig. 6

Ans. Area of equilateral triangle of side 14 cm $= \left(\frac{\sqrt{3}}{4} \times 196\right) cm^2$

$= 49 \times \sqrt{3}$ Or $49 \times 1.73 cm^2$

= 84.77 cm^2

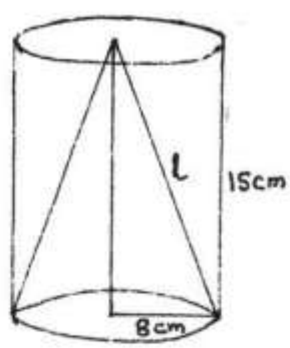
Combined area of three sectors $= \left(\frac{180}{360} \times \frac{22}{7} \times 7 \times 7\right) cm^3$

= 77 cm^2

$\therefore$ Shaded area = (84.77 – 77.00) = 7.77 cm^2

33. From a solid cylinder whose height is 15 cm and diameter 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. [Take π =3.14]

Ans.



r = 8 cm, h = 15 cm

$\therefore l = \left(\sqrt{8^2 + 15^2}\right) cm = 17 cm$

∴ Total surface area of remaining solid

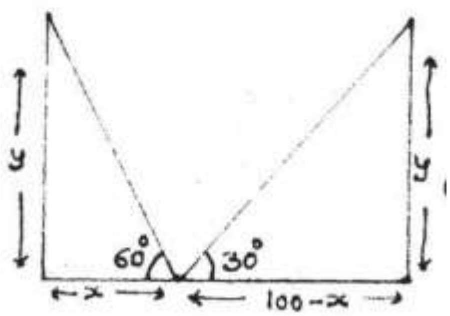
$= 3.14\,[2rh + r^2 + rl] = 3.14 \times 8\,[30 + 8 + 17]$

$= 3.14 \times 8 \times 55\ \text{cm}^2$

$= 1381.6\ \text{cm}^2$

34. Two people of equal height are standing opposite of the each other on either side of the road, which is 100 m wide. From a point between them on the road, the angles of eleventh of the top of the poles are 60° and 30°, respectively. Find the height of the poles.

Ans.



(i) $\frac{y}{x} = \tan 60^\circ = \sqrt{3}$

$\Rightarrow x = \frac{y}{\sqrt{3}}$

(ii) $\frac{y}{100 - x} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

$100 - \frac{y}{\sqrt{3}} = \sqrt{3}y$

$\Rightarrow 100\sqrt{3} = 4y \Rightarrow y = 25\sqrt{3}$

$\therefore$ Height of each pole $= 25\sqrt{3}$ m